1 式の計算

A

1

(1)

$$\begin{aligned}
与式 &= \{(x-2)(x+1)\}\{(x-4)(x+3)\}+24 \\
&= \{(x^2-x)-2\}\{(x^2-x)-12\}+24 \\
&= (x^2-x)^2-14(x^2-x)+48 \\
&= \{(x^2-x)-6\}\{(x^2-x)-8\} \\
&= (x^2-x-6)(x^2-x-8) \\
&= (x+2)(x-3)(x^2-x-8)
\end{aligned}$$

(2)

$$\begin{aligned}
与式 &= (y+z+3)x+y^2+yz+5y+2z+6 \\
&= (y+z+3)x+(y+2)z+y^2+5y+6 \\
&= (y+z+3)x+(y+2)z+(y+2)(y+3) \\
&= (y+z+3)x+(y+2)\{z+(y+3)\} \\
&= (y+z+3)x+(y+2)(y+z+3) \\
&= \{x+(y+2)\}(y+z+3) \\
&= (x+y+2)(y+z+3)
\end{aligned}$$

補足

次数の小さい文字式順に整理すると楽。

つまり，与式は x の 1 次式，z の 1 次式，y の 2 次式だから，

まず x または z について整理

上のように x について整理した場合は，次に，y と z の式の部分を z について整理，

そして，y の 2 次式を因数分解

(3)

$$\begin{aligned}
与式 &= (a^2-2ab+b^2)c+(b^2-2bc+c^2)a+(c^2-2ca+a^2)b+8abc \\
&= (b+c)a^2+(b^2+2bc+c^2)a+bc(b+c) \\
&= (b+c)a^2+(b+c)^2a+bc(b+c) \\
&= (b+c)\{a^2+(b+c)a+bc\} \\
&= (b+c)\{(a+b)(a+c)\} \\
&= (a+b)(b+c)(c+a)
\end{aligned}$$

(4)

$$
\begin{aligned}
\text{与式} &= \{(x+y)(x+z)\}(y+z)+xyz \\
&= \{x^2+(y+z)x+yz\}(y+z)+xyz \\
&= (y+z)x^2+\{(y+z)^2+yz\}x+yz(y+z) \\
&= \{x+(y+z)\}\{(y+z)x+yz\} \\
&= (x+y+z)(xy+yz+zx)
\end{aligned}
$$

(5)

$$
\begin{aligned}
\text{与式} &= a^4-2(b^2+c^2)a^2+b^4-2b^2c^2+c^4 \\
&= a^4-2(b^2+c^2)a^2+(b^2-c^2)^2 \\
&= \{a^2-(b^2+c^2)\}^2-(b^2+c^2)^2+(b^2-c^2)^2 \\
&= (a^2-b^2-c^2)^2-4b^2c^2 \\
&= (a^2-b^2-c^2)^2-(2bc)^2 \\
&= \{(a^2-b^2-c^2)-2bc\}\{(a^2-b^2-c^2)+2bc\} \\
&= \{a^2-(b^2+2bc+c^2)\}\{a^2-(b^2-2bc+c^2)\} \\
&= \{a^2-(b+c)^2\}\{a^2-(b-c)^2\} \\
&= \{a+(b+c)\}\{a-(b+c)\}\{a+(b-c)\}\{a-(b-c)\} \\
&= (a+b+c)(a-b-c)(a+b-c)(a-b+c)
\end{aligned}
$$

2

(1)

$$
\begin{aligned}
\text{与式} &= \frac{2}{(1+\sqrt{2})+\sqrt{3}}\cdot\frac{(1+\sqrt{2})-\sqrt{3}}{(1+\sqrt{2})-\sqrt{3}}+\sqrt{\frac{4-2\sqrt{3}}{2}} \\
&= \frac{2(1+\sqrt{2}-\sqrt{3})}{(1+\sqrt{2})^2-(\sqrt{3})^2}+\frac{\sqrt{(\sqrt{3}-1)^2}}{\sqrt{2}} \\
&= \frac{2(1+\sqrt{2}-\sqrt{3})}{2\sqrt{2}}+\frac{\sqrt{3}-1}{\sqrt{2}} \\
&= \frac{1+\sqrt{2}-\sqrt{3}}{\sqrt{2}}+\frac{\sqrt{3}-1}{\sqrt{2}} \\
&= 1
\end{aligned}
$$

(2)

$$与式=\frac{1+\sqrt{2}+i}{\left(1+\sqrt{2}\right)-i}\cdot\frac{\left(1+\sqrt{2}\right)+i}{\left(1+\sqrt{2}\right)+i}$$
$$=\frac{\left\{\left(1+\sqrt{2}\right)+i\right\}^2}{\left(1+\sqrt{2}\right)^2-i^2}$$
$$=\frac{\left(1+\sqrt{2}\right)^2+2\left(1+\sqrt{2}\right)i+i^2}{3+2\sqrt{2}+1}$$
$$=\frac{3+2\sqrt{2}+2\left(1+\sqrt{2}\right)-1}{4+2\sqrt{2}}$$
$$=\frac{2\left(1+\sqrt{2}\right)+2\left(1+\sqrt{2}\right)i}{2\sqrt{2}\left(\sqrt{2}+1\right)}$$
$$=\frac{1+i}{\sqrt{2}}$$
$$=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$$

よって，実部は$\frac{\sqrt{2}}{2}$，虚部は$\frac{\sqrt{2}}{2}$

3

(1)

$$与式=\frac{1}{1-\frac{1}{\frac{-x}{1-x}}}$$
$$=\frac{1}{1+\frac{1-x}{x}}$$
$$=\frac{1}{\frac{1}{x}}$$
$$=x$$

(2)

$$
\begin{aligned}
\text{与式} &= 1+\frac{2}{x}-\frac{(x+1)+2}{x+1}-\frac{(x-3)-2}{x-3}+\frac{(x-4)-2}{x-4} \\
&= 1+\frac{2}{x}-\left(1+\frac{2}{x+1}\right)-\left(1-\frac{2}{x-3}\right)+1-\frac{2}{x-4} \\
&= 2\left(\frac{1}{x}-\frac{1}{x+1}+\frac{1}{x-3}-\frac{1}{x-4}\right) \\
&= 2\left\{\frac{(x+1)-x}{x(x+1)}+\frac{(x-4)-(x-3)}{(x-3)(x-4)}\right\} \\
&= 2\left\{\frac{1}{x(x+1)}-\frac{1}{(x-3)(x-4)}\right\} \\
&= 2\cdot\frac{-8x+12}{x(x+1)(x-3)(x-4)} \\
&= -\frac{8(2x-3)}{x(x+1)(x-3)(x-4)}
\end{aligned}
$$

補足

分子の式の次数を分母の式の次数より小さくすることから始める。

4

$\omega^2+\omega+1=0$ より，$\omega(\omega^2+\omega+1)=0$

また，$\omega^2+\omega=-1$ だから，

$$
\begin{aligned}
\omega(\omega^2+\omega+1) &= \omega^3+\omega^2+\omega \\
&= \omega^3-1
\end{aligned}
$$

よって，$\omega^3-1=0$ すなわち $\omega^3=1$

これより，

$$
\begin{aligned}
\text{与式} &= (a+b+c)\{a+b\omega+c(-\omega-1)\}\{a+b(-\omega-1)+c\omega^3\cdot\omega\} \\
&= (a+b+c)\{(a-c)+(b-c)\omega\}\{a-b(\omega+1)+c\omega\} \\
&= (a+b+c)\{(a-c)+(b-c)\omega\}\{(a-b)-(b-c)\omega\} \\
&= (a+b+c)\left[(a-c)(a-b)+\{(a-b)-(a-c)\}(b-c)\omega-(b-c)^2\omega^2\right] \\
&= (a+b+c)\{(a-c)(a-b)-(b-c)^2\omega-(b-c)^2\omega^2\} \\
&= (a+b+c)\{(a-c)(a-b)-(b-c)^2(\omega^2+\omega)\} \\
&= (a+b+c)\{(a-c)(a-b)-(b-c)^2\cdot(-1)\} \\
&= (a+b+c)\{(a-c)(a-b)+(b-c)^2\} \\
&= (a+b+c)(a^2+b^2+c^2-ab-bc-ca) \\
&= a^3+b^3+c^3-3abc
\end{aligned}
$$

5

(1)

$$\text{与式}=\left(x^4+1\right)^2-2x^4$$
$$=\left(x^4+1\right)^2-\left(\sqrt{2}x^2\right)^2$$
$$=\left\{\left(x^4+1\right)+\sqrt{2}x^2\right\}\left\{\left(x^4+1\right)-\sqrt{2}x^2\right\}$$
$$=\left\{\left(x^2+1\right)^2-\left(2-\sqrt{2}\right)x^2\right\}\left\{\left(x^2+1\right)^2-\left(2+\sqrt{2}\right)x^2\right\}$$
$$=\left\{\left(x^2+1\right)^2-\left(\sqrt{2-\sqrt{2}}\,x\right)^2\right\}\left\{\left(x^2+1\right)^2-\left(\sqrt{2+\sqrt{2}}\,x\right)^2\right\}$$
$$=\left\{\left(x^2+1\right)+\sqrt{2-\sqrt{2}}\,x\right\}\left\{\left(x^2+1-\sqrt{2-\sqrt{2}}\,x\right)\right\}\left\{\left(x^2+1\right)+\sqrt{2+\sqrt{2}}\,x\right\}\left\{\left(x^2+1\right)-\sqrt{2+\sqrt{2}}\,x\right\}$$
$$=\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)\left(x^2+\sqrt{2+\sqrt{2}}\,x+1\right)\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)$$

(2)

$$\text{与式}=(x+y)^3-3xy(x+y)+1-3xy$$
$$=(x+y)^3+1^3-3xy\{(x+y)+1\}$$
$$=\{(x+y)+1\}^3-3(x+y)\cdot1\{(x+y)+1\}-3xy(x+y+1)$$
$$=(x+y+1)^3-3(x+y)(x+y+1)-3xy(x+y+1)$$
$$=(x+y+1)\left\{(x+y+1)^2-3(x+y)-3xy\right\}$$
$$=(x+y+1)\left(x^2+y^2-xy-x-y+1\right)$$

補足： $a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)$

(3)

$$\text{与式}=(x-y)^3+(z-y)^3-\{(x-y)+(z-y)\}^3$$
$$=(x-y)^3+(z-y)^3-\left\{(x-y)^3+(z-y)^3+3(x-y)(z-y)\{(x-y)+(z-y)\}\right\}$$
$$=3(x-y)(z-y)(x-2y+z)$$

6

(1)

$$\text{与式}=\left[\left(9+4\sqrt{5}\right)^n+\left(9-4\sqrt{5}\right)^n+\left\{\left(9+4\sqrt{5}\right)^n-\left(9-4\sqrt{5}\right)^n\right\}\right]$$
$$\times\left[\left(9+4\sqrt{5}\right)^n+\left(9-4\sqrt{5}\right)^n-\left\{\left(9+4\sqrt{5}\right)^n-\left(9-4\sqrt{5}\right)^n\right\}\right]$$
$$=4\left(9+4\sqrt{5}\right)^n\left(9-4\sqrt{5}\right)^n$$
$$=4\left\{\left(9+4\sqrt{5}\right)\left(9-4\sqrt{5}\right)\right\}^n$$
$$=4\cdot1^n$$
$$=4$$

(2)

$$与式=\left\{\left(\frac{1+i}{\sqrt{2}}\right)^2\right\}^p+\left\{\left(\frac{1-i}{\sqrt{2}}\right)^2\right\}^p$$

$$=i^p+(-i)^p$$

$$=i^p+(-1)^p i^p$$

p は奇数だから，$(-1)^p=-1$

よって，

$$与式=i^p-i^p$$

$$=0$$

(3)

$$与式=-\frac{a^3}{(a-b)(c-a)}-\frac{b^3}{(a-b)(b-c)}-\frac{c^3}{(c-a)(b-c)}$$

$$=-\frac{a^3(b-c)+b^3(c-a)+c^3(a-b)}{(a-b)(b-c)(c-a)}$$

$$=-\frac{(b-c)a^3-(b^3-c^3)a+b^3c-bc^3}{(a-b)(b-c)(c-a)}$$

$$=-\frac{(b-c)a^3-(b-c)(b^2+bc+c^2)a+bc(b+c)(b-c)}{(a-b)(b-c)(c-a)}$$

$$=-\frac{(b-c)\{a^3-(b^2+bc+c^2)a+bc(b+c)\}}{(a-b)(b-c)(c-a)}$$

$$=-\frac{(b-c)(a^3-ab^2-abc-ac^2+b^2c+bc^2)}{(a-b)(b-c)(c-a)}$$

$$=-\frac{(b-c)\{(c-a)b^2+(c^2-ca)b+a^3-ac^2\}}{(a-b)(b-c)(c-a)}$$

$$=-\frac{(b-c)\{(c-a)b^2+c(c-a)b-a(c-a)(c+a)\}}{(a-b)(b-c)(c-a)}$$

$$=-\frac{(b-c)(c-a)\{b^2+bc-a(c+a)\}}{(a-b)(b-c)(c-a)}$$

$$=-\frac{(b-c)(c-a)\{-(a-b)c-(a^2-b^2)\}}{(a-b)(b-c)(c-a)}$$

$$=\frac{(b-c)(c-a)\{(a-b)c+(a-b)(a+b)\}}{(a-b)(b-c)(c-a)}$$

$$=\frac{(b-c)(c-a)(a-b)(c+a+b)}{(a-b)(b-c)(c-a)}$$

$$=a+b+c$$

7

$$\left(3+2\sqrt{2}\right)^4-\left\{1-\left(3-2\sqrt{2}\right)^4\right\}=\left(3+2\sqrt{2}\right)^4+\left(3-2\sqrt{2}\right)^4-1$$
$$=\left\{\left(3+2\sqrt{2}\right)^2+\left(3-2\sqrt{2}\right)^2\right\}^2-2\left(3+2\sqrt{2}\right)^2\left(3-2\sqrt{2}\right)^2-1$$
$$=34^2-2\left\{\left(3+2\sqrt{2}\right)\left(3-2\sqrt{2}\right)\right\}^2-1$$
$$=1156-2\cdot1-1$$
$$=1153$$

より，

$$\left(3+2\sqrt{2}\right)^4=1153+\left\{1-\left(3-2\sqrt{2}\right)^4\right\}\quad\cdots①$$

$2^2<8<3^2$ より， $2<2\sqrt{2}<3$

したがって， $-3<-2\sqrt{2}<-2$

これより， $3+(-3)<3+\left(-2\sqrt{2}\right)<3+(-2)$ すなわち $0<3-2\sqrt{2}<1$

よって， $0<\left(3-2\sqrt{2}\right)^4<1$

ゆえに， $0<1-\left(3-2\sqrt{2}\right)^4<1\quad\cdots②$

①，②より， $\left(3+2\sqrt{2}\right)^4$ の小数部分は $1-\left(3-2\sqrt{2}\right)^4$ である。